

Contents

Foreword	xv
Abstract	xvii
<i>Volver</i>	xix
Acknowledgments	xxi
Introduction	1
Outline of the monograph	3
I Preliminary Concepts	7
1 Universal Algebra	9
2 Deductive Systems	19
3 Gentzen Systems	31
II Substructural Systems	41
4 Basic Intuitionistic Substructural Systems	43
4.1 The Calculi \mathbf{FL}_σ with Negations	43
4.2 Mirror Images	48
4.3 Characterization of the Leibniz Congruence	50
4.4 Hilbert-style Axiomatization for Systems $\epsilon\mathcal{FL}_\sigma$	54

4.4.1	Strongly Separable Axiomatizations	56
5	Definability of Connectives and Definitional Expansions	59
5.1	Motivation and definitions	59
5.2	Definability and Definitional Expansions.	62
5.3	About calculi with exchange rules	66
5.4	Definability of Zero in \mathbf{FL}_σ	70
5.5	Definability of the negations in \mathbf{FL}_σ	71
5.6	Definability of 1	73
5.7	Versions without negations	73
5.7.1	Equivalence of the two versions for calculi \mathbf{FL}_σ	74
III	Fragments without Implication	77
6	Basic Ordered Algebraic Structures	79
6.1	Order-Algebras	79
6.2	Partially Ordered Monoids and Groupoids	81
6.3	The varieties $\mathring{\mathbf{M}}_\sigma^{sl}$ and $\mathring{\mathbf{M}}_\sigma^\ell$	83
6.4	Residuated Structures	89
7	Adding Negation Operators: Pseudocomplemented Structures	101
7.1	The operations of pseudocomplementation	101
7.2	Characterization of the class \mathbf{PM}^\preceq	106
7.3	The classes \mathbf{PM}^{sl} and \mathbf{PM}^ℓ are varieties	107
7.4	Pseudocomplementation with respect to the minimum	109
7.5	Weakly contractive pseudocomplemented monoids	113
7.6	Involutive pseudocomplemented monoids	116
8	Completions and Subreducts	123
8.1	Basic concepts	123
8.2	Completions	125
8.2.1	Ideal-completion of $\mathring{\mathbf{M}}^{sl}$ -algebras and \mathbf{FL} -algebras	127
8.2.2	Dedekind-MacNeille completion for \mathbf{FL}_σ -algebras	131

8.2.3	The DM -completion does not work for $\mathring{M}^{s\ell}$ -algebras	135
8.2.4	Comparison between the two completions in \mathbb{FL} -algebras	139
8.2.5	On the minimality of the DM -completion	144
8.3	Reducts and subreducts	147
9	Algebraic Analysis of some Implication-free Fragments	151
9.1	Algebraization	151
9.2	Fragments without implications.	160
9.3	Some results on decidability	165
9.4	On some systems with weak contraction	166
10	Three Implication-free Fragments of t-Norm Based Fuzzy Logics	169
10.1	Analysis of the fragments	170
11	Conclusions and Future Work	177
	Alphabetic Index	181
	Symbols Index	187
	Bibliography	191