

# Contents

<b>Foreword</b>	<b>xv</b>
<b>Abstract</b>	<b>xvii</b>
<b>Acknowledgments</b>	<b>xix</b>
<b>1 Introduction</b>	<b>1</b>
1.1 Context . . . . .	1
1.2 Objectives . . . . .	4
1.3 Contributions . . . . .	5
1.4 Publications . . . . .	6
1.5 Overview . . . . .	8
<b>2 Algorithms for SAT and MAX-SAT</b>	<b>11</b>
2.1 Definitions . . . . .	11
2.2 SAT algorithms . . . . .	13
2.2.1 Resolution . . . . .	13
2.2.2 The Davis-Putnam procedure . . . . .	13
2.2.3 The Davis-Logemann-Loveland procedure . . . . .	15
2.2.4 Local search procedures for SAT . . . . .	17
2.2.5 Overview of SAT algorithms . . . . .	19
2.3 MAX-SAT algorithms . . . . .	25
2.3.1 Branch and Bound . . . . .	26
2.3.2 Local search and approximation algorithms for MAX-SAT . . . . .	28
2.3.3 Overview of BnB algorithms for MAX-SAT . . . . .	29
2.3.4 Solvers submitted to the MAX-SAT Evaluation 2006 . . . . .	33
2.4 Summary . . . . .	34
<b>3 Lower Bounds</b>	<b>35</b>
3.1 Related work . . . . .	35
3.2 Star rule . . . . .	38
3.3 Lower Bound UP . . . . .	39
3.3.1 Understanding the lower bound through the implication graph . . . . .	40

3.3.2	Implementing the lower bound UP . . . . .	42
3.4	UP improved: Choosing the best unit clause . . . . .	43
3.4.1	Lower bounds improving UP . . . . .	43
3.4.2	Extending lower bound UP with Failed Literal Detection . . . . .	50
3.5	Empirical evaluation . . . . .	51
3.5.1	Benchmarks . . . . .	51
3.5.2	Experimental results . . . . .	52
3.6	Summary . . . . .	55
<b>4</b>	<b>Inference rules</b>	<b>65</b>
4.1	Related work . . . . .	66
4.2	UP based inference rules . . . . .	68
4.2.1	Integer programming transformation of a CNF formula . . . . .	69
4.2.2	Inference rules . . . . .	69
4.3	On implementing the inference rules . . . . .	73
4.3.1	Complexity, termination, and (in)completeness of the applications of the rules . . . . .	75
4.4	Experimental results . . . . .	76
4.5	Summary . . . . .	88
<b>5</b>	<b>Implementing a weighted MAX-SAT solver</b>	<b>91</b>
5.1	Basic equivalences for weighted MAX-SAT . . . . .	92
5.2	Lazy solver . . . . .	92
5.2.1	Data structures . . . . .	93
5.2.2	Variable selection heuristic . . . . .	94
5.3	Empirical evaluation . . . . .	96
5.3.1	Benchmarks . . . . .	96
5.3.2	Experimental results . . . . .	97
5.4	Summary . . . . .	106
<b>6</b>	<b>Empirical comparison of MAX-SAT and weighted MAX-SAT</b>	<b>109</b>
6.1	Solvers . . . . .	109
6.1.1	Other existing MAX-SAT solvers . . . . .	109
6.1.2	Our contribution . . . . .	111
6.2	Experimentation on MAX-SAT . . . . .	112
6.3	Experimentation on weighted MAX-SAT . . . . .	113
6.4	Summary . . . . .	116
<b>7</b>	<b>Conclusions</b>	<b>125</b>
<b>A</b>	<b>Additional inference rules</b>	<b>127</b>
A.1	Unit clause creation rules . . . . .	127
	<b>Bibliography</b>	<b>129</b>

# List of Figures

2.1	Search tree for DLL applied to Example 2.4. . . . .	17
2.2	Search tree for MAX-SAT BnB applied to Example 2.5. . . . .	29
3.1	Created implication graph for Example 3.5 applying lower bound UP. The dotted area contains the conflict graph. . . . .	45
3.2	Created implication graphs for Example 3.5 applying lower bound $UP^S$ . Both graphs correspond to the conflict graphs. . . . .	46
3.3	Created implication graphs for Example 3.5 applying lower bound $UP^*$ . Both graphs correspond to the conflict graphs. . . . .	46
3.4	Implication graph for Example 3.6. The dotted area contains the conflict graph nodes detected by $UP^S$ ; and the dashed area contains the conflict graph nodes detected by $UP^*$ . . . . .	47
3.5	Implication graph for Example 3.7. The dotted area contains the conflict graph nodes detected by $UP^S$ and $UP^*$ . . . . .	49
3.6	Impact of heuristics UP, $UP^*$ and $UP^S$ . . . . .	53
3.7	Impact of failed literal detection on heuristics UP, $UP^*$ and $UP^S$ . . . . .	54
3.8	Impact of failed literal detection on heuristics UP, $UP^*$ and $UP^S$ . . . . .	56
3.9	Random MAX-2-SAT with 50 variables . . . . .	57
3.10	Random MAX-2-SAT with 100 variables . . . . .	58
3.11	Random MAX-3-SAT with 50 variables . . . . .	59
3.12	Random MAX-3-SAT with 70 variables . . . . .	60
3.13	Impact of heuristics UP, $UP^*$ and $UP^S$ on MAX-CUT . . . . .	61
3.14	Impact of failed literal detection on heuristics UP, $UP^*$ and $UP^S$ on MAX-CUT . . . . .	61
3.15	Impact of failed literal detection on heuristics UP, $UP^*$ and $UP^S$ in MAX-CUT . . . . .	62
3.16	Random MAX-CUT with 50 variables . . . . .	63
4.1	Random MAX-2-SAT with 50 variables . . . . .	77
4.2	Random MAX-2-SAT with 100 variables . . . . .	78
4.3	Random MAX-3-SAT with 50 variables . . . . .	79
4.4	Random MAX-3-SAT with 70 variables . . . . .	80
4.5	Random MAX-CUT with 50 variables . . . . .	81
4.6	Random MAX-2-SAT 50 variables . . . . .	82
4.7	Random MAX-2-SAT 100 variables . . . . .	83

4.8	Random MAX-3-SAT 50 variables . . . . .	84
4.9	Random MAX-3-SAT 70 variables . . . . .	85
4.10	MAX-CUT . . . . .	86
5.1	Comparison of applying the first phase only and the two phases in the variable selection heuristic. . . . .	95
5.2	Weighted Random MAX-2-SAT 50 variables . . . . .	99
5.3	Weighted Random MAX-2-SAT 100 variables . . . . .	100
5.4	Weighted Random MAX-3-SAT 50 variables . . . . .	101
5.5	Weighted Random MAX-3-SAT 70 variables . . . . .	102
5.6	Random Graph Coloring . . . . .	103
5.7	Random MAX-ONES 2-SAT . . . . .	104
5.8	Random MAX-ONES 3-SAT . . . . .	105
6.1	Random MAX-2-SAT solver comparison . . . . .	114
6.2	Random MAX-2-SAT with 150 variables solver comparison . . .	116
6.3	Random MAX-3-SAT solver comparison . . . . .	117
6.4	Random MAX-CUT solver comparison . . . . .	119
6.5	Random weighted MAX-2-SAT solver comparison . . . . .	120
6.6	Random weighted MAX-3-SAT solver comparison . . . . .	121
6.7	Graph coloring solver comparison . . . . .	122
6.8	MAX-ONES solver comparison . . . . .	123

# List of Tables

2.1	Execution track of a BnB for Example 2.5. . . . .	28
4.1	Rule evaluation by benchmarks in the MAX-SAT Evaluation 2006.	90
4.2	Rule evaluation by benchmarks in the MAX-SAT Evaluation 2006 with failed literal detection . . . . .	90
5.1	Evaluation results for the seven solvers . . . . .	107
6.1	MAX-SAT solvers from other research works. . . . .	111
6.2	MAX-SAT solvers we have implemented . . . . .	112
6.3	Experimental results for all the unweighted benchmarks in the MAX-SAT Evaluation 2006. . . . .	115
6.4	Experimental results for all the weighted benchmarks in the MAX- SAT Evaluation 2006. . . . .	118